Insights from Linear Predictions of Aircraft Response to Damaged Airfields

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This paper examines the dynamic response of a simplified mathematical model of an aircraft that taxies over two arbitrary disturbances. It produces the idea of the bump multiplier that determines explicitly whether the second discrete disturbance will amplify or attenuate the response from the first disturbance. While the assumptions are very severe, the results can be useful to gain physical insight, to guide more elaborate nonlinear calculations, and to plan test programs.

Introduction

THE problem of aircraft dynamic response to taxiing over rough surfaces has been a subject of analysis and test for many years. For the most part, the work has been limited to predicting and/or measuring the dynamic response of an aircraft due to the (nearly) random roughness of the terrain or by wear and tear on runways and taxiways. Within the last several years, however, concerns have arisen within the defense agencies of the NATO countries about the safety of aircraft operations over the discrete disturbances which can arise from bomb-damaged and repaired runways.

As a result of those concerns the U.S. Air Force instituted Program HAVE BOUNCE[1,2] that performs flight (taxi) tests over simulated, relatively mild runway damage and repairs for several USAF combat and transport aircraft. HAVE BOUNCE also develops computer programs[3] to predict the dynamic response to the simulated runway profiles. Other NATO nations are performing similar test and analysis programs on their aircraft.

HAVE BOUNCE considers the computer programs to be validated when they produce satisfactory comparisons with the experimental results from flight (taxi) tests under relatively mild conditions. Then HAVE BOUNCE uses the validated computer programs to extrapolate from the test conditions to more severe operational cases.

Because the taxi test programs have proven to be very expensive and difficult to repeat exactly, the USAF also created the Aircraft Ground-Induced Loads Excitation (AGILE)[4] facility that measures the dynamic response of operational aircraft to damaged and repaired runways within the controlled conditions of the laboratory. AGILE supports an operational aircraft on its tires on massive hydraulic shakers and drives the shakers vertically to represent the vertical events of the aircraft taxiing over damaged and repaired runways.

Each of the three integrated shakers can sustain a static weight of 50,000 lb, displace amplitudes of 10 in., impose dynamic forces up to 50,000 lb, and be driven sinusoidally (frequencies up to 25 Hz), randomly, or to follow prescribed discrete motions. At present, AGILE is limited to providing solely vertical dynamic forces to the aircraft (no lateral or drag forces or rotating wheels). In its first major test, agreement between the AGILE tests and HAVE BOUNCE taxi tests for an operational A-7D aircraft was excellent.

All three evaluation methods—computer programs, HAVE BOUNCE tests, and AGILE tests on operational aircraft—have been dominated by one major consideration: the nonlinearities in the landing gear. As a result, nearly all of the computations have been done with numerical time-integration of the nonlinear differential equations of motion. The taxi tests and AGILE tests also have been forced to adopt a tedious approach of repetitive, trial-and-error test cases, again because concerns over strong nonlinearities prevented the consideration of the superposition of simple disturbances to synthesize more complex responses.

In this paper, we contend that the nonlinearities do indeed strongly influence the computational and test results, especially the exact levels of the loads obtained. However, the qualitative response, the physical understanding, and the selection of speeds, bump heights, and bump spacings which produce large dynamic responses ought to be predictable for the most part by simpler linear methods. Nonlinear calculations, taxi tests, and AGILE tests ought to be preceded by linearized calculations that can be done rapidly and can yield much physical insight into those conditions that produce extensive dynamic response. A clever analyst may be able to find the simplicity and intuitive understanding in seemingly complex time histories, which in fact may be not much more than superpositions of many relatively simple events.

The purpose of this paper is to show how linear analyses with a one-degree-of-freedom model can yield an understanding of complex time histories and how they can be used to plan nonlinear calculations, taxi tests, and AGILE tests. The paper illustrates the principles by treating the response of a linear one-degree-of-freedom oscillator as it taxies over two successive discrete disturbances.

Clearly, this simplified approach should be extended to include a second degree of freedom such as aircraft pitch and to include the possibility of excitation through a second landing gear. Those are straightforward extensions which have indeed been performed, but we omit them here to illustrate only the main ideas.

Single-Degree-of-Freedom Oscillator to a Single Disturbance

Assume a single-degree-of-freedom oscillator, with damping less than the critical value, receives some excitation from an arbitrary disturbance, but that the excitation stops at time $t = t_1$. For example, Fig. 1 illustrates the acceleration of such a system in response to a triangular disturbance, ending at $t = t_1$.

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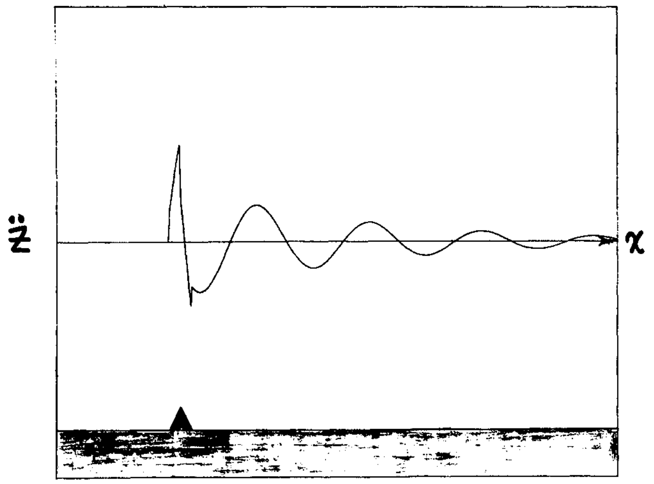

Fig. 1 Acceleration response to a single triangular disturbance.

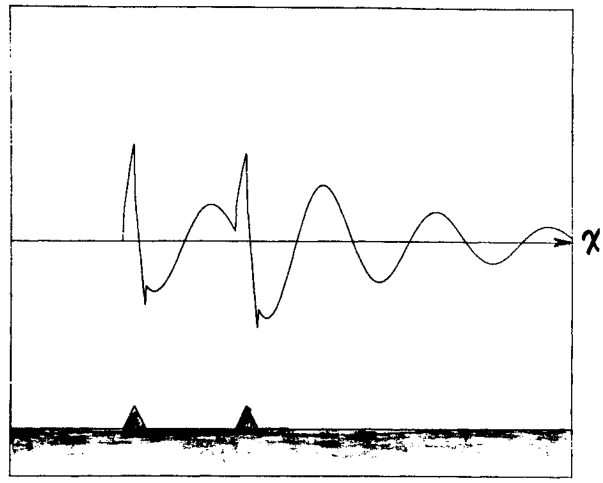

Fig. 2 Acceleration response to two triangular disturbances.

For times after t_1, when the response is decaying freely, the acceleration response can be written:

$$\ddot{z}(t)|_{t>t_1} = e^{-\alpha\omega(t-t_1)}\{A_1 \sin[\omega(t-t_1)] + B_1 \cos[\omega(t-t_1)]\} \quad (1)$$

where

t_1 = the time the excitation ends,
$\alpha\omega$ = damping,
ω = damped frequency, and
A_1, B_1 = constants which depend upon α, ω, the excitation and the initial conditions.

Note that the damping parameter α above is not quite the same as ζ, the frequently used fraction of critical damping from the classical, single-degree-of-freedom oscillator. At this point we merely want to observe that the response is damped and oscillatory and that it could come from test data or analyses.

The decaying acceleration response also can be written as:

$$\ddot{z}(t)|_{t>t_1} = R_1 e^{-\alpha\omega(t-t_1)} \sin[\omega(t-t_1)+\phi_1] \quad (2)$$

where

$$R_1 = \sqrt{A_1^2 + B_2^2}$$

$$\tan\phi_1 = B_1/A_1$$

We loosely refer to R as the potential amplitude of the acceleration response. It is an upper bound (not necessarily the best one) on the amplitude of the acceleration response to a single disturbance. The phase shift ϕ_1 depends only on A_1 and B_1 and will therefore be different for different excitations and initial conditions.

By differentiating Eq. (2) with respect to time, we see that the locally extreme values of $\ddot{z}(t)|_{t>t_1}$ will occur at the values of time for which

$$\omega(t-t_1) = (2n-1)\frac{\pi}{2} - (\phi_1 + \delta); \qquad n = 1,2,3,\ldots \quad (3)$$

where

$$\tan\delta = \alpha$$

The additional phase shift δ will be small for values of damping that are small with respect to the critical value, ($\alpha \ll 1$). Note, however, that the first phase shift ϕ_1 need not be small.

Two Successive Disturbances

Now suppose the oscillator receives a subsequent arbitrary excitation over another period of time, and that excitation stops at time $t = t_2$. For example, Fig. 2 illustrates the acceleration response to two successive triangular disturbances, the second ending at $t = t_2$.

If there had been no previous disturbance, the acceleration response to the second disturbance would have been

$$\ddot{z}(t)|_{t>t_2} = e^{-\alpha\omega(t-t_2)}\{A_2 \sin[\omega(t-t_2)] + B_2 \cos[\omega(t-t_2)]\} \quad (4)$$

However, because of the first disturbance, the acceleration response to the combined disturbances actually is

$$\ddot{z}(t)|_{t>t_2} = e_1(A_1S_1 + B_1C_1) + e_2(A_2S_2 + B_2C_2) \quad (5)$$

where

$$e_i = e^{-\alpha\omega(t-t_i)}$$

$$S_i = \sin[\omega(t-t_i)]$$

$$C_i = \cos[\omega(t-t_i)]$$

The trick is to write Eqs. (4) and (5) with respect to the time of the most recent disturbance, t_2. To that end we write

$$t - t_1 = (t-t_2) + (t_2 - t_1)$$

$$e_{21} = e^{-\alpha\omega(t_2-t_1)}$$

$$S_{21} = \sin[\omega(t_2-t_1)]$$

$$C_{21} = \cos[\omega(t_2-t_1)]$$

The acceleration response to the combined disturbances then can be rewritten as

$$\ddot{z}(t)|_{t>t_2} = R_2 e^{-\alpha\omega(t-t_2)} \sin[\omega(t-t_2)+\phi_2] \quad (6)$$

where

$$R_2 = \sqrt{\begin{aligned}&(A_2^2 + B_2^2) + 2e_{21}[C_{21}(A_1A_2 + B_1B_2)\\ &+ S_{21}(A_1B_2 - B_1A_2)] + e_{21}^2(A_1^2 + B_1^2)\end{aligned}} \quad (7)$$

$$\tan\phi_2 = \frac{B_2 + e_{21}(A_1 S_{21} + B_1 C_{21})}{A_2 + e_{21}(A_1 C_{21} - B_1 S_{21})} \tag{8}$$

Equation (7) for R_2, the potential amplitude of the acceleration response to the combined disturbances, is one of the most useful findings of this paper. Much of the subsequent work here will be concerned with finding the conditions which locally maximize or minimize R_2.

Similarly to the results for the single disturbance, the locally extreme values of $\ddot{z}(t)|_{t>t_2}$ will occur when

$$\omega(t - t_2) = (2n-1)\frac{\pi}{2} - (\phi_2 + \delta); \qquad n = 1,2,3,\ldots \tag{9}$$

We have seen how to find the times for locally extreme values of the decaying acceleration response, assuming we know A_1, B_1, A_2, B_2, α, ω, t_1, and t_2. However, we actually are searching for the best and worst possible runway profiles, so we want to find the values of t_2 that will give locally extreme values of the potential amplitude R_2. We differentiate Eq. (7) with respect to t_2 and set the result to zero to obtain

$$\tilde{R}_{12}\sin[\omega(t_2 - t_1) + \psi_{12}] + \frac{\alpha}{\sqrt{1+\alpha^2}} e^{-\alpha\omega(t_2 - t_1)} = 0 \tag{10}$$

where

$$\tilde{R}_{12} = \sqrt{\frac{A_2^2 + B_2^2}{A_1^2 + B_1^2}} \tag{11}$$

$$\tan\psi_{12} = \frac{\alpha(A_1A_2 + B_1B_2) - (A_1B_2 - B_1A_2)}{(A_1A_2 + B_1B_2) + \alpha(A_1B_2 - B_1A_2)} \tag{12}$$

The exact solution for the time delays $(t_2 - t_1)$ that give the locally extreme values of R_2 would require a numerical or graphical solution of Eq. (10). However, for small damping we would expect

$$\omega(t_2 - t_1) \approx n\pi - \psi_{12}; \qquad n = 1,2,3\ldots \tag{13}$$

Bump Multiplier

Recall that R_1 represented the potential amplitude of the decaying acceleration response to the first disturbance and that R_2 represented the potential amplitude of the response to the combined disturbances, where in each case we measured time from the time of the most recent disturbance. We call the ratio R_2/R_1 the *bump multiplier*, since it defines the extent to which the second disturbance amplifies or attenuates the acceleration response to the first disturbance. The *bump multiplier* is

$$\frac{R_2}{R_1} = \sqrt{\frac{(A_2^2 + B_2^2) + 2e_{21}[C_{21}(A_1A_2 + B_1B_2) + S_{21}(A_1B_2 - B_1A_2)] + e_{21}^2(A_1^2 + B_1^2)}{A_1^2 + B_1^2}} \tag{14}$$

To assist in the interpretation of the *bump multiplier* we add another set of abbreviations

$$\epsilon_i = \frac{B_i}{A_i}; \qquad i = 1,2$$

to obtain

$$\frac{R_2}{R_1} = \left|\frac{A_2}{A_1}\right| \sqrt{\frac{(1+\epsilon_2^2) + 2\left(\frac{A_1}{A_2}e_{21}\right)[C_{21}(1+\epsilon_1\epsilon_2) + S_{21}(\epsilon_2 - \epsilon_1)] + \left(\frac{A_1}{A_2}e_{21}\right)^2(1+\epsilon_1^2)}{1+\epsilon_1^2}} \tag{15}$$

When we use these abbreviations to find the time delays $(t_2 - t_1)$ that give the locally extreme values of the potential amplitude R_2, we find that $t_2 - t_1$ must satisfy the following:

$$\left|\frac{A_2}{A_1}\right| \sqrt{\frac{1+\epsilon_2^2}{1+\epsilon_1^2}} \sin[\omega(t_2 - t_1) + \psi_{12}] + \frac{\alpha}{\sqrt{1+\alpha^2}} e^{-\alpha\omega(t_2 - t_1)} = 0 \tag{16}$$

where

$$\tan\psi_{12} = \frac{\alpha(1+\epsilon_1\epsilon_2) - (\epsilon_2 - \epsilon_1)}{(1+\epsilon_1\epsilon_2) + \alpha(\epsilon_2 - \epsilon_1)} \tag{17}$$

Use of the Average Speed

We have made no assumption of a constant taxi speed between the two disturbances. If l is the distance between the two disturbances, the average speed is

$$\tilde{V} = \frac{1}{t_2 - t_1} \tag{18}$$

Then we can express the term

$$\omega(t_2 - t_1) = \frac{l\omega}{\tilde{V}} = \tilde{\lambda} \tag{19}$$

The usual terminology for $\lambda = l\omega/V$, based on the instantaneous speed, is the reduced frequency. Therefore $\tilde{\lambda} = l\omega/\tilde{V}$ is the reduced frequency based on the average speed between the two disturbances.

Special Case: Similar Disturbances

We now define similar disturbances as discrete disturbances that have the same shape but differ only in magnitude and/or sign. Examples would be the entire family of infinite ramps or a family of sine waves of the same wavelength but varying heights. The assumption of similar disturbances is not a very limiting one. In fact, nearly all of the profiles tested in the HAVE BOUNCE program and all of the NATO/AGARD profiles can be broken down into sequences of similar ramp disturbances. For linear systems with zero initial conditions, similar disturbances will produce similar acceleration responses, and when disturbances are similar, $\epsilon_1 = \epsilon_2 = \epsilon$.

Under the assumption of similar disturbances, the potential amplitudes and phase angles become

$$R_1 = |A_1|\sqrt{1+\epsilon^2} \tag{20}$$

$$R_2 = |A_2|\sqrt{1+\epsilon^2}\sqrt{1 + 2\left(\frac{A_1}{A_2}e_{21}\right)C_{21} + \left(\frac{A_1}{A_2}e_{21}\right)^2} \tag{21}$$

$$\tan\phi_1 = \epsilon \tag{22}$$

$$\tan\phi_2 = \frac{\epsilon + \left(\frac{A_1}{A_2}e_{21}\right)(S_{21} + \epsilon C_{21})}{1 + \left(\frac{A_1}{A_2}e_{21}\right)(C_{21} - \epsilon S_{21})} \tag{23}$$

$$\tan\psi_{12} = \alpha \tag{24}$$

The *bump multiplier* is

$$\frac{R_2}{R_1} = \left|\frac{A_2}{A_1}\right|\sqrt{1 + 2\left(\frac{A_1}{A_2}e_{21}\right)C_{21} + \left(\frac{A_1}{A_2}e_{21}\right)^2} \tag{25}$$

Figures 3 and 4 illustrate the *bump multiplier* for equal and opposite disturbances as a function of the reduced frequency based on the average speed.

The major conclusions to be drawn are as follows:

1) The spacing that maximizes or minimizes the acceleration response depends very weakly on the damping parameter, α, and

2) The first approximation for the best/worst reduced frequencies,

$$\tilde{\lambda} \approx n\pi - \psi_{12} \approx n\pi - \alpha$$

is an excellent approximation to the exact solution for reasonably small values of damping.

Example: Spring-Mass-Damper-Taxiing Over Two Ramps

We consider the example of a classical, single-degree-of-freedom oscillator that encounters two ramp disturbances. The disturbances are separated by a distance l and occur at times t_1 and t_2, respectively. The taxi speeds V_1, V_2 are not necessarily equal at the time of the encounters, nor are the ramp angles θ_1, θ_2.

The differential equation of motion is:

$$m\ddot{z} + c\dot{z} + kz = c\dot{g} + kg \tag{26}$$

where

$$g(t) = V\theta t u(t)$$

$$u(t) = \text{unit step function}$$

We make the usual abbreviations:

$$\zeta = \text{ratio of damping to critical value, } c/2m\omega_0$$

$$\omega_0 = \text{undamped natural frequency, } \sqrt{k/m}$$

By solving the ordinary differential equation for the displacement in response to infinite ramp inputs (with zero initial conditions), and then differentiating those results twice with respect to time, we find the various parameters to use in Eqs. (1) and (4):

$$A_i = V_i\theta_i\omega_0(1 - 2\zeta^2) \tag{27a}$$

$$B_i = 2V_i\theta_i\omega_0\zeta\sqrt{1-\zeta^2} \tag{27b}$$

$$\omega = \text{damped frequency, } \omega_0\sqrt{1-\zeta^2} \tag{27c}$$

$$\alpha = \zeta/\sqrt{1-\zeta^2} \tag{27d}$$

For purposes of illustration we pick the fictitious undamped natural frequency to be

$$f_0 = \omega_0/2\pi = 1.0\,\text{Hz}$$

and we pick the damping value

$$\zeta = 0.1$$

All members of the family of infinite ramps are similar. Therefore, for every ramp input (regardless of speed V, frequency ω, or slope θ) the similarity parameter ϵ is

$$\epsilon = \frac{B}{A} = \frac{2\zeta\sqrt{1-\zeta^2}}{1-2\zeta^2} = \tan\phi_1 \approx 0.2030 \tag{28}$$

The second phase shift for the location of the locally extreme values of the decaying acceleration response will be

$$\delta = \sin^{-1}\zeta = 0.1002 \text{ rad} \approx 5.739 \text{ deg}$$

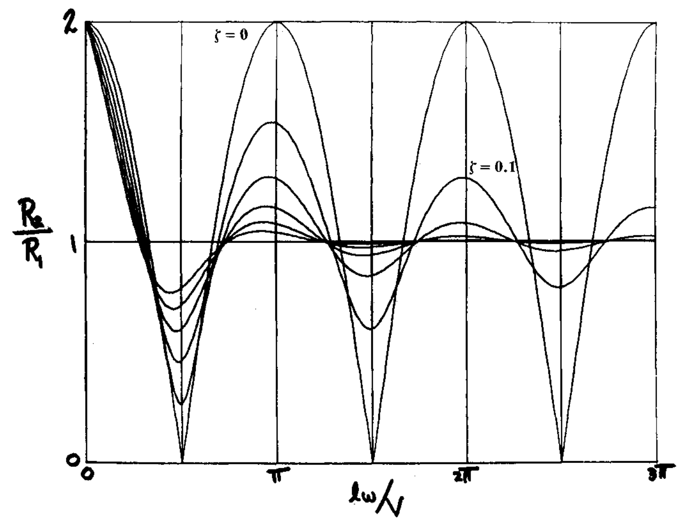

Fig. 3 The *bump multiplier* for equal disturbances.

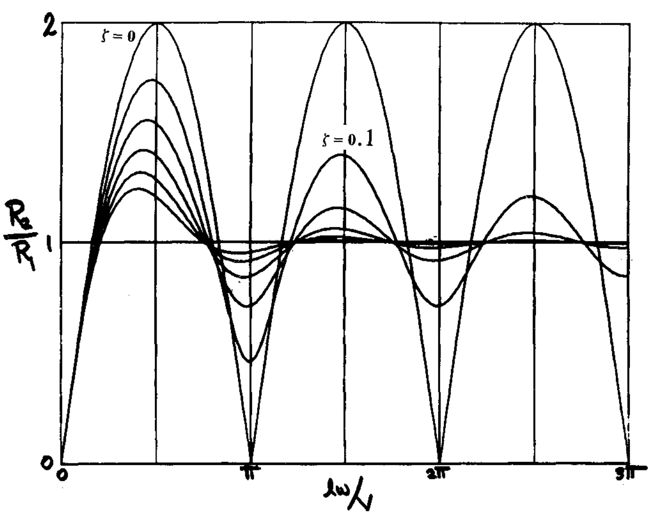

Fig. 4 The *bump multiplier* for opposite disturbances.

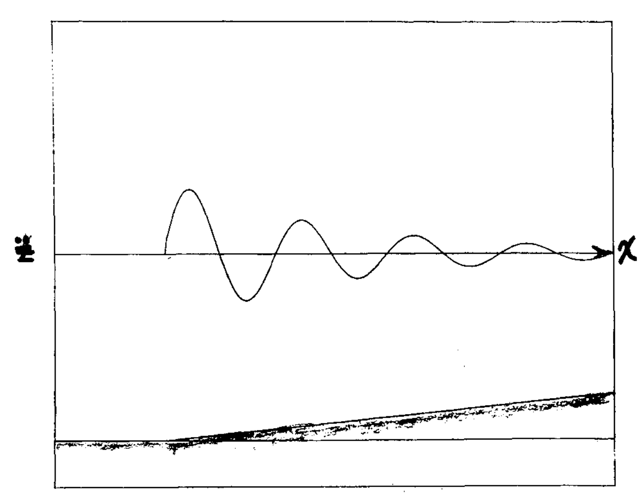

Fig. 5 Acceleration response to an infinite ramp.

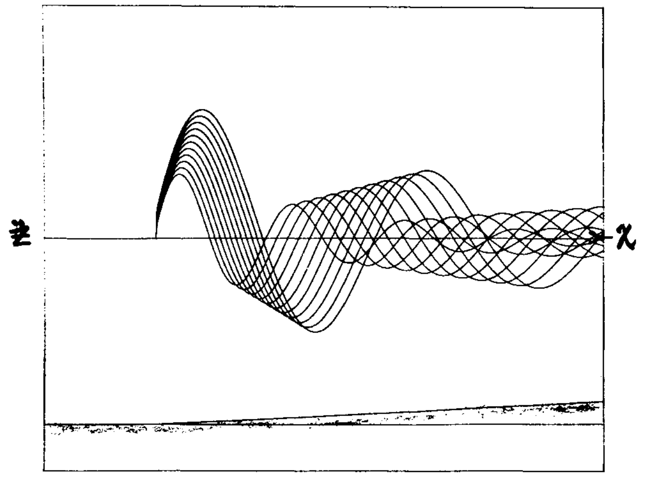

Fig. 6 Acceleration response to an infinite ramp at several speeds.

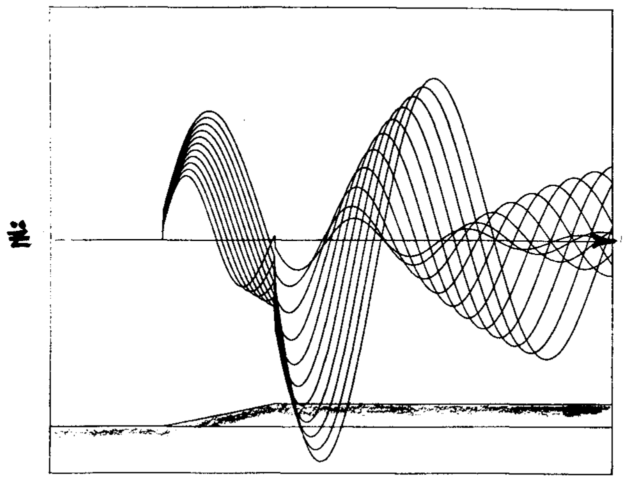

Fig. 8 Acceleration response to two infinite ramps at several speeds.

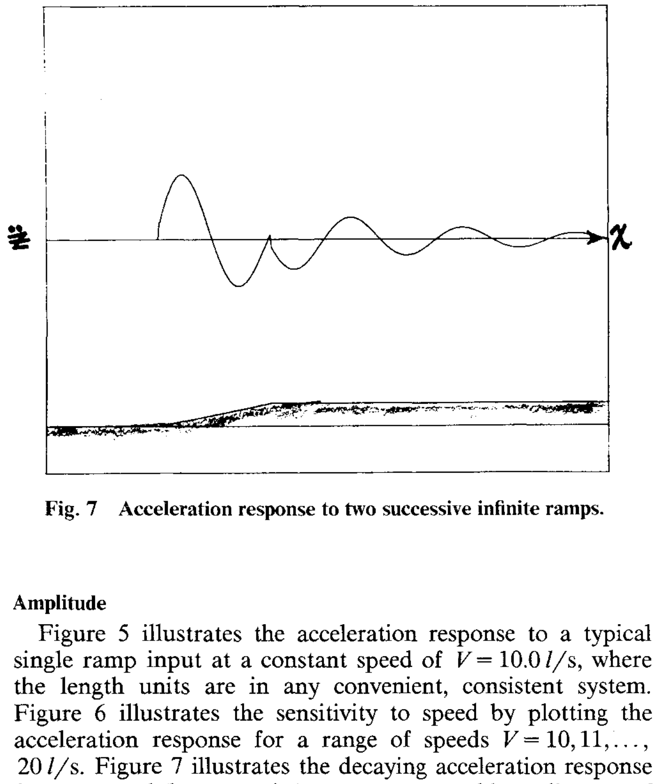

Fig. 7 Acceleration response to two successive infinite ramps.

Amplitude

Figure 5 illustrates the acceleration response to a typical single ramp input at a constant speed of $V = 10.0\, l/\mathrm{s}$, where the length units are in any convenient, consistent system. Figure 6 illustrates the sensitivity to speed by plotting the acceleration response for a range of speeds $V = 10, 11, \ldots, 20\, l/\mathrm{s}$. Figure 7 illustrates the decaying acceleration response for two equal (but opposite) ramps, separated by a distance of $20.0\, l$, at a constant speed of $10.0\, l/\mathrm{s}$. Figure 8 illustrates the sensitivity to speed for the range of speeds $V = 10, 11, \ldots, 20\, l/\mathrm{s}$. While the amplitude of the acceleration response to the single disturbance grew monotonically with increasing speed, the amplitude of the acceleration response to the combined disturbances displays a much more complicated structure.

Bump Multiplier

Reverting to the general case of nonconstant speeds and ramp angles, the potential amplitude for the dynamic acceleration response to the combined disturbances is given by

$$R_2 = \omega_0 \sqrt{(V_2\theta_2)^2 + 2(V_1\theta_1 e_{21})C_{21} + (V_1\theta_1 e_{21})^2} \qquad (29)$$

The *bump multiplier* is

$$\frac{R_2}{R_1} = \left|\frac{V_2\theta_2}{V_1\theta_1}\right| \sqrt{1 + 2\left(\frac{V_1\theta_1 e_{21}}{V_2\theta_2}\right)C_{21} + \left(\frac{V_1\theta_1 e_{21}}{V_2\theta_2}\right)^2} \qquad (30)$$

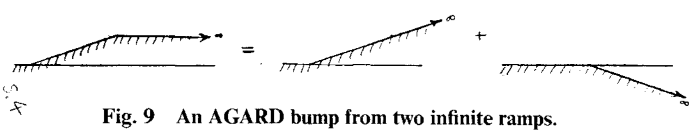

Fig. 9 An AGARD bump from two infinite ramps.

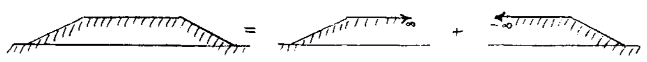

Fig. 10 An AGARD repair mat from two AGARD bumps.

The dominant term is $C_{21} = \cos\tilde{\lambda}$, which is modified by $V_1\theta_1 e_{21}/V_2\theta_2$.

Equations (29) and (30) are very powerful results that relate the potential amplitude R_2, and the *bump multiplier* to the instantaneous speeds V_1, V_2, the average speed $\tilde{V}$, the ramp angles θ_1, θ_2, the damping parameter α, and the average reduced frequency $\tilde{\lambda} = l\omega/\tilde{V}$.

Best/Worst Runway Profiles

The first approximation to the time delays that locally maximize and minimize the acceleration response to the combined disturbances is:

$$\lambda \approx n\pi - \psi_{12}; \qquad n = 1, 2, 3 \ldots$$

where

$$\tan\psi_{12} = \alpha \approx 0.1005$$

or

$$\frac{\lambda}{n\pi} = 0.9681, 0.9841, 0.9894, 0.9920\ldots; \qquad n = 1, 2, 3, 4\ldots$$

Application to Nonlinear Calculations and Test Programs

Three Principles

The first set of basic ideas to keep in mind when using these results to plan nonlinear calculations, HAVE BOUNCE (taxi) tests, or AGILE tests is that a useful building block is the infinite ramp, that two infinite (opposite) ramps can combine to produce an AGARD bump, and that two (opposite) AGARD bumps can combine to produce an AGARD repair mat (Figs. 9 and 10).

Second, recall that for two disturbances separated by a distance l, the best/worst combinations tend to occur when

$$\tilde{\lambda} = \frac{l\omega}{\tilde{V}} = 2\pi\frac{lf}{\tilde{V}} \approx n\pi - \psi_{12} \approx n\pi - \alpha; \qquad n = 1,2,3\ldots$$

Third, the potential amplitudes of the acceleration response of a classical single-degree-of-freedom oscillator to a single infinite ramp and to two combined ramps are

$$R_1 = V_1\omega_0|\theta_1|$$

$$R_2 = \omega_0\sqrt{(V_2\theta_2)^2 + 2(V_2\theta_2)(V_1\theta_1 e_{21})C_{21} + (V_1\theta_1 e_{21})^2} \quad (31)$$

Obtaining the Infinite Ramp Data from the Test Results for an AGARD Bump

Because of the impossibility of experimentally developing an infinite ramp, it will be more practical to excite the oscillator with an AGARD bump and then infer what the acceleration response would have been to an infinite ramp. Since the acceleration response will undoubtedly not be purely in a single-degree-of-freedom, we must process the test data to obtain separate values of R_2, α, ω, and ϕ_2 for each degree of freedom in the equation:

$$\ddot{z}(t)|_{t>t_2} = R_2 e^{-\alpha\omega(t-t_2)}\sin[\omega(t-t_2) + \phi_2]$$

For each degree of freedom we will also have e_{21}, S_{21}, and C_{21} to obtain the potential amplitude R_1 from:

$$R_1 = \frac{R_2}{\sqrt{1 - 2e_{21}C_{21} + e_{21}^2}} \quad (32)$$

A good test of our assumed linearity is to form $R_1/V\theta$. The values for each degree of freedom should be approximately independent of speed V or angle θ.

Finally, the phase lag ϕ_1 can be obtained from:

$$Q = \frac{e_{21}S_{21}}{1 - e_{21}C_{21}} \quad (33)$$

$$\epsilon = \frac{\tan\phi_2 + Q}{1 - Q\tan\phi_2} \quad (34)$$

$$\tan\phi_1 = \epsilon$$

Guidelines for Nonlinear Calculations, AGILE Tests, and Taxi Tests

We begin by calculating or measuring the acceleration response to an AGARD bump (Fig. 11) over a range of speeds V and angles θ.

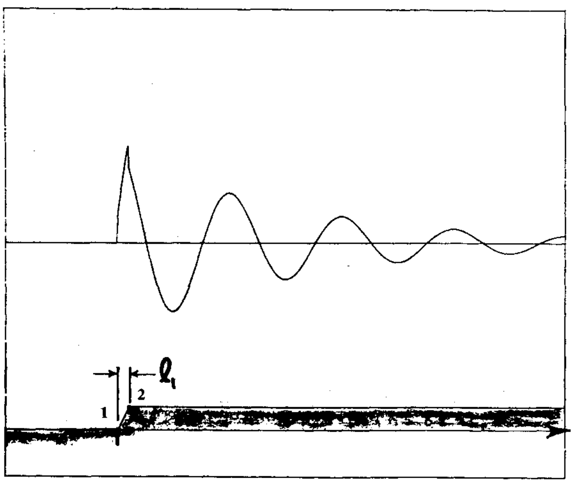

Fig. 11 Geometry of an AGARD bump.

The next step should be to test the linear result that the best/worst AGARD bumps will be those for which

$$\tilde{\lambda} = \frac{l_1\omega}{V} \approx n\pi - \alpha; \qquad n = 1,2,3\ldots$$

Since l_1 is fixed by the AGARD geometry, we can accomplish this variation by choosing the speeds to be:

$$V \approx \frac{l_1\omega}{n\pi - \alpha}, \qquad n = 1,2,3\ldots$$

We can interpret the final slope of the AGARD repair mat (Fig. 12) between points x_3 and x_4 as just the negative of the initial slope; with the only distinction being that it begins at a distance of $l_1 + l_2$ after the initial slope. Then we can search for the best/worst length of the repair mat by setting:

$$\tilde{\lambda} = \frac{(l_1 + l_2)\omega}{\tilde{V}} \approx n\pi - \alpha; \qquad n = 1,2,3\ldots$$

where $\tilde{V}$ is the average speed over the distance between x_1 and x_3. In this case, we have both the average speed $\tilde{V}$ and the average length l_2 to use as variables.

Now we note that the total length of the AGARD repair mat is $2l_1 + l_2$ and assume that another repair mat is placed a distance l_3 behind the first mat (Fig. 13). Therefore, to look

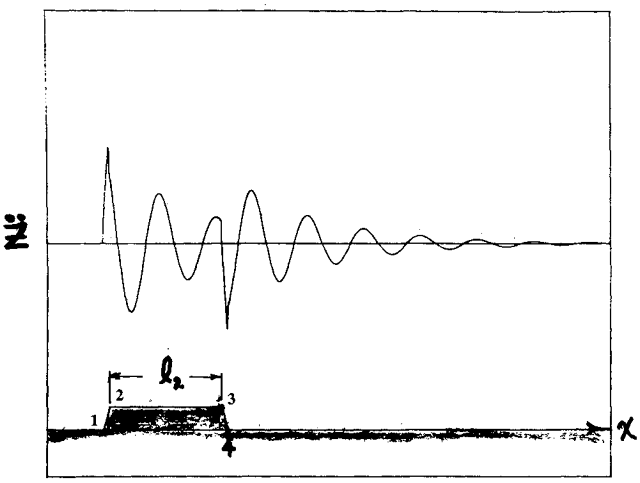

Fig. 12 Geometry of an AGARD repair mat.

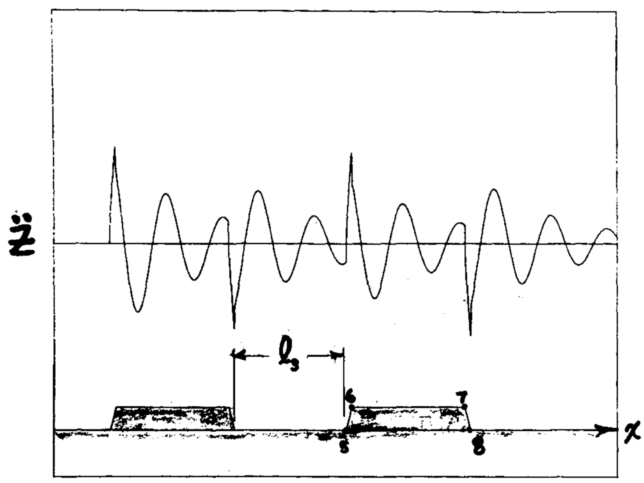

Fig. 13 Geometry of two AGARD repair mats.

for the best/worst spacings we set:

$$\tilde{\lambda} = \frac{(2l_1 + l_2 + l_3)\,\omega}{\tilde{V}} \approx n\pi - \alpha; \qquad n = 1,2,3\ldots$$

where $\tilde{V}$ is the average speed between the two repair mats.

Conclusions

We have treated the dynamic response of an aircraft taxiing over runway disturbances under the assumption that the gross aspects of the dynamic response can be found in the analysis of a linear, one-degree-of-freedom system excited by two successive disturbances. We have found the following:

1) There is a great deal that can be learned about the governing physics for aircraft dynamic response to taxi over damaged and repaired runways by examining the results of calculations with relatively simple, linear models.

2) The seemingly complicated time histories can be merely superpositions of relatively simple, time-phased events.

3) Relatively simple expressions are available for the potential amplitude (an upper bound) of the acceleration response excited by one or two disturbances. In the (not too) special case of similar, disturbances separated by a distance l, with nonconstant speeds and ramp angles, the expression for the potential amplitude R_2 is:

$$R_2 = \omega_0\sqrt{(V_2\theta_2)^2 + 2V_2\theta_2(V_1\theta_1 e_{21})C_{21} + (V_1\theta_1 e_{21})^2}$$

where $e_{21} = e^{-\alpha\tilde{\lambda}}$, $C_{21} = \cos\tilde{\lambda}$, and $\tilde{\lambda} = l\omega/\tilde{V}$.

4) The effects of disturbance spacing and variable taxi speed are controlled by the reduced frequency, based on the average speed between disturbances.

5) One need not actually calculate the time histories to find the best/worst profiles and speeds, but can use the expressions for the potential amplitude R_2 and the bump multiplier R_2/R_1.

6) To maximize/minimize dynamic response, a good approximation for $\tilde{\lambda}$ is $\tilde{\lambda} \approx n\pi - \alpha$.

7) While damping obviously controls the dynamic response to the disturbances, the critical speeds and disturbance spacings are weak functions of damping.

8) These results can easily be extended from two disturbances to an arbitrary number of disturbances and multiple-degree-of-freedom systems with multiple landing gear.

9) The results of calculations based on these linear methods should be compared with results from flight (taxi) tests, AGILE tests, and nonlinear calculations. This is not to say that the linear results should be relied upon to predict detailed loads; rather the question should be, "Do the simple linear models predict the critical speeds and spacings so that we can use them to guide our test programs and nonlinear solutions?"

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